Differential Geometry, Analysis and Epistemology in Milan - a conference in honor of Marco Rigoli's 70th birthday

Luis J. Alìas (Universidad de Murcia)

Maximum principles with geometric applications

Abstract: in this lecture we will introduce some new forms of maximum principles on complete, non-compact Riemannian manifolds, with applications to different geometric situations like, for instance, Bernstein-type results for entire graphs or applications to capillary and prescribed mean curvature graphs. Our results are part of our joint work with Antonio Caminha and F. Yure do Nascimento, and, more recently, with Marco Rigoli and Giulio Colombo.

Luca Brandolini (Università degli Studi di Bergamo)

Single Radius Spherical Cap Discrepancy on Compact Two-Point Homogeneous Spaces

Abstract: I will discuss some new results on the single radius spherical discrepancy in the context of compact two-point homogeneous spaces. Consider $\mathcal{M}$ as such a space, and let d its real dimension. Let $\{z_j\}_{j=1}^N$ be a finite sequence of points inside $\mathcal{M}$. We define the discrepancy for a ball $B_r(x)$ centred at x of radius r as

$$D_r(x) = \sum_{j=1}^{N} \chi_{B_r(x)}(z_j) - N\,|B_r(x)|\,.$$

Our main result establishes conditions on r such that, for some positive constant C, independent of the choice of points $\{z_j\}_{j=1}^N$ the following inequality holds:

$$\int_{\mathcal{M}} |D_r(x)|^2\,dx \geqslant CN^{1-1/d}.$$

These conditions on r depends on the dimension d. Interestingly, they cannot be met when d is congruent to 1 modulo 4. Still, in those cases, we are able to prove a somewhat weaker estimate.

Giovanni Catino (Politecnico di Milano)

Stable minimal hypersurfaces *via* conformal methods and applications

Abstract: In this talk I will describe some recent results concerning the rigidity of complete, immersed, stable (or delta-stable) minimal hypersurfaces in the Euclidean space. The results rely on a conformal method, inspired by classical papers of Schoen-Yau and Fischer-Colbrie. I will also present several applications of these techniques.

Giulio Colombo (Università degli Studi di Milano)

Minimal graphs and nonnegative Ricci curvature

Abstract: we discuss some recent results concerning uniqueness and rigidity of solutions to the minimal surface equation on complete Riemannian manifolds of nonnegative Ricci curvature, including a Liouville type theorem for entire positive solutions and some splitting and "almost-splitting" results in presence of nonconstant solutions of bounded gradient or linear growth. This is based on joint works with E. S. Gama, M. Magliaro, L. Mari and M. Rigoli.

Leonardo Colzani (Università degli Studi di Milano-Bicocca)

Pick's Theorem and the Euler-MacLaurin summation formula

Abstract: we present a Fourier analytic proof of Pick's theorem on the area of a polygon with integer vertices in terms of the number of integer points inside and on its boundary. We also hint at a connection with a multidimensional Euler-MacLaurin summation formula. Joint work with Luca Brandolini, Sinai Robins, Giancarlo Travaglini.

Alberto Farina (Université de Picardie Jules Verne)

Classification Results, Rigidity Theorems and semilinear PDEs on Riemannian Manifolds: a P-Function Approach

Abstract: We consider solutions to critical and sub-critical semilinear elliptic PDEs on complete, noncompact Riemannian manifolds and study their classification as well as the effect of their presence on the underlying manifold. When the Ricci curvature is nonnegative, we prove both the classification of positive solutions to the critical equation and the rigidity for the ambient manifold. The same results are established for solutions to the Liouville equation on Riemannian surfaces. Our results are obtained via an appropriate

P-function whose constancy implies the classification of both the solutions and the underlying manifold.

Roberta Filippucci (Università degli Studi di Perugia)

Liouville Type Results for Problems with Gradient Terms

Abstract: A priori estimates can generally be categorized as uniform estimates, integral estimates, or pointwise estimates. In this talk, we present some recent results dealing with some a priori estimates for positive solutions to elliptic quasilinear problems, involving both equations and inequalities, with a reaction depending on the gradient which leads to a lack of variational structure. Finally, the strict connection between Liouville type results and pointwise a priori estimates will be discussed.

Anna Fino (Università di Torino)

An Overview on strong Geometries with Torsion

Abstract: A strong geometry with torsion is a Riemannian manifold carrying a metric connection with closed skew-symmetric torsion. In the seminar I will first review general properties of metric connections with closed skew-symmetric torsion. Then I will focus on the case of Hermitian manifolds and 7-manifolds endowed with a G_2-structure.

Luigi Fontana (Università degli Studi di Milano-Bicocca)

Sobolev borderline Estimates on Manifolds with Euclidean Volume Growth

Abstract: in the Euclidean space the critical case of Sobolev embeddings is fairly well understood in terms of sharp exponential inequalities of the Moser-Trudinger-Adams type. The same problem, when the ambient space is a Riemannian manifold, faces a variety of issues depending on the geometry (locally and globally). In this talk we concentrate on the manifolds with nonnegative Ricci curvature and Euclidean volume growth. Joint work with Carlo Morpurgo and Liuyu Qin.

Gary Jensen (Washington University in St. Louis)

Möbius Geometry by Moving Frames

Abstract: Möbius geometry is the unit sphere $\mathbb{S}^{n+1}$ under the action of the Lie group G of all conformal diffeomorphisms of it. The group G is isomorphic to $\mathbf{SO}(n+1,1)$ and acts transitively on $\mathbb{S}^{n+1}$. We use the standard frame reduction of the method of moving frames to give an elementary construction of the Möbius first and second fundamental forms of C.P. Wang and of an elementary proof of Wang's congruence theorem. The exceptional nature of the case $n = 2$ is also treated.

Albino Lanciani (C.R.E.A.L.P.)

Matematica e Velocità (ovvero una delle cose che mi ha insegnato Marco)

Abstract: come si può descrivere e quale senso logico ha l'attività di un matematico allorché dimostra o costituisce una qualche forma di oggettualità? Quando il matematico dimostra qualcosa – e costituisce propriamente un oggetto – cosa succede da un punto di vista logico? E psicologico? Qual è il carattere fenomenologico del vissuto nel quale appunto si compie quest'operazione?

Marco Magliaro (Università degli Studi dell'Insubria)

Sharp pinching theorems for complete CMC submanifolds in the sphere

Abstract: in 1968 Simons proved that if a compact, minimal submanifold of the unit sphere $f : M^n \to \mathbb{S}^{n+p}$ has second fundamental form satisfying $|A|^2 \leq np/(2p-1)$, then either $|A| \equiv 0$ and M is a great sphere, or $|A|^2 \equiv np/(2p-1)$. Lawson and Chern, do Carmo & Kobayashi characterized the latter case and proved that if $|A|^2 \equiv np/(2p-1)$, then M is a Clifford torus or a Veronese surface. This pinching theorem was later generalized by Alencar & do Carmo for compact CMC hypersurfaces of the sphere and by Santos for compact PMC submanifolds of the sphere. In this talk we extend the results by Simons, Lawson, Chern, do Carmo & Kobayashi and Alencar & do Carmo to complete submanifolds of the sphere. We also partially generalize the result of Santos in dimension $n \leq 6$. This is joint work with L. Mari, F. Roing and A. Savas-Halilaj.

Andrea Malchiodi (Scuola Normale Superiore)

Yamabe Metrics on conical Manifolds

Abstract: We prove existence of Yamabe metrics on singular manifolds with conical points and conical links of Einstein type that include orbifold structures. We deal with metrics of generic type and derive a counterpart of Aubin's classical result. The singular nature of the metric determines a different condition on the dimension, compared to the regular case.We derive asymptotic expansions on the Yamabe quotient by adding a proper and implicit lower-order correction to standard bubbles, whose contribution to the expansion of the quotient can be determined combining the decomposition of symmetric 2-tensor fields and Fourier analysis on the conical links. Some perspectives for other singular manifolds will be discussed. This is joint work with Mattia Freguglia and Francesco Malizia.

Carlo Mantegazza (Università degli Studi di Napoli Federico II)

The Riemannian Penrose inequality *via* nonlinear Potential Theory

Abstract: I will discuss the Riemannian Penrose inequality in an asymptotically flat 3-manifold with nonnegative scalar curvature and the main points of a new proof by means of a monotonicity formula holding along the level sets of the p-capacitary potentials of the horizon of a black hole. Joint work with Virginia Agostiniani, Lorenzo Mazzieri and Francesca Oronzio.

Dario Monticelli (Politecnico di Milano)

Liouville theorems for critical degenerate elliptic equations

Abstract: in this talk I will present some classification results of positive solutions to the well-known critical Laplace equation

$$-\Delta u = u^{2^*-1},$$

on Riemannian manifolds with nonnegative Ricci curvature.

I will also discuss some recent classification results for positive solutions to the CR Yamabe equation on the Heisenberg group $\mathbb{H}^n$

$$-\Delta_b u = u^{2_*-1},$$

where Δ_b denotes the sub-Laplacian operator on $\mathbb{H}^n$, with particular attention to the case $n = 1$.

Finally I will discuss a generalization of this result to the CR Yamabe equation on noncompact $(2n+1)$-dimensional Sasakian manifolds with nonnegative curvature.

This is based on recent joint works with G. Catino, A. Roncoroni (Politecnico di Milano), Y.Y. Li (Rutgers University) and X. Wang (Michigan State University).

Barbara Nelli (Università degli Studi dell'Aquila)

Special Weingarten Surfaces with convex Boundary

Abstract: an oriented surface Σ immersed in $\mathbb{R}^3$ is called a *Special Weingarten surface* if its principal curvatures κ_1, κ_2 satisfy a relation

$$W(\kappa_1, \kappa_2) = 0, \tag{1}$$

where $W : \mathbb{R}^2 \to \mathbb{R}$ is a function of class C^1 and

$$\frac{\partial W}{\partial \kappa_1}\frac{\partial W}{\partial \kappa_2} > 0, \tag{2}$$

holds on the subset of $\mathbb{R}^2$ given by $W^{-1}(\{0\})$. The class of Special Weingarten surfaces clearly includes surfaces with constant mean curvature and surfaces with positive Gaussian curvature. In the 1950's, A.D. Alexandrov, H. Hopf, S. Chern, P. Hartman & A. Wintner investigated closed Special Weingarten surfaces, extending results known for constant mean curvature surfaces. We aim to study the topology and geometry of Special Weingarten surfaces with convex planar boundary. This is joint work with Giuseppe Pipoli and Marcos Paulo Tassi.

Fabrizio Palombi (Università della Calabria)

Gian-Carlo Rota tra fenomenologia, matematica e scienza

Abstract: l'intervento presenterà alcuni dei principali contributi originali di Rota alla riflessione fenomenologica dedicando particolare attenzione alla sua ricerca sulla Fundierung ovvero sulla relazione di fondazione strutturale introdotta da Husserl nelle terza delle Ricerche logiche. Sarà considerata, in particolare, l'indagine di Rota sulle modalità con le quali la matematica struttura la scienza e costruisce i suoi oggetti ideali evidenziando le differenze fenomenologiche tra l'esperienza e l'esperimento e tra la conoscenza comune e quella scientifica.

Stefano Pigola (Università degli Studi di Milano-Bicocca)

Volume and parabolicity for drifted Laplacians

Abstract: shrinking Ricci solitons and properly immersed self-shrinkers of the mean curvature flow have finite weighted volumes. The first claim is a traditional consequence of volume comparisons whereas the second one is a beautiful remark by Q. Ding and Y. L. Xin in 2011 and relies on integral inequalities on the level sets of the potential function. Soon after, based on similar arguments, X. Cheng and D. Zhou detected a set of differential inequalities on the potential proper function under which a weighted manifold has finite weighted volume. This gave a unified function-theoretic proof of the volume properties alluded to above. In this talk I will consider the Cheng-Zhou type result under a quite different viewpoint that is suitable to be extended to general drifted Laplacians. Within this broader context, we are naturally led to speculate about parabolicity and to what extent it is related to volumes. It is a work in progress joint with Leandro Pessoa and Alberto G. Setti.

Patrizia Pucci (Università degli Studi di Perugia)

On certain critical elliptic Equations and System in $\mathbb{R}^N$

Abstract: the talk presents some recent results on existence of solutions for double critical equations in $\mathbb{R}^N$. The first part deals with the existence of nontrivial nonnegative solutions of model parametric elliptic equations in $\mathbb{R}^N$, involving a possibly supercritical term in the Sobolev sense, and a nonlocal term with the upper Hardy-Littlewood-Sobolev critical exponent. Under some conditions, we describe the precise parametric range of existence and nonexistence of a nonnegative solution. The last part of the talk shows qualitative properties, existence and non-existence results of solutions of k-coupled Hardy-Hénon elliptic systems in $\mathbb{R}^N$, obtained with different proof techniques. Finally, some open problems are also given.

Isabel Salavessa (Instituto Superior Técnico, Universidade de Lisboa)

New examples of Riemannian Domains satisfying Pólya Conjecture on Laplacian Eigenvalues

Abstract: In 1954, George Pólya conjectured that on an Euclidean domain Ω the Dirichlet (Neumann, respectively) eigenvalues and the first term of the corresponding Weyl asymptotics satisfy two inequalities respectively, giving a proof in the case of Ω tiles the entire

Euclidean space $\mathbb{R}^n$. Few other examples have been showed until recently in 2023 Filonov Et al. in [FLPS] proved the conjecture holds for the Euclidean disks. At around the same time, in [FS] we obtained the existence of large families that satisfy the conjecture, both in Euclidean and Riemannian manifolds with some symmetries, like sufficiently small sectors of geodesic disks or of convex domains of revolution, and cylindrical surfaces with small height. In [FMS] we also show the Neumann eigenvalues of the n-dimensional hemisphere $\mathbb{S}^n_+$ satisfy Pólya inequality but not the Dirichlet ones if $n \geq 3$, not even eventually. On the other hand a Pólya inequality with a correction constant second term holds, that corresponds to an arithmetic and geometric means type inequality. Furthermore, in [FS-2025] we also show that two among the three classes of lunes of $\mathbb{S}^n_+$ the Dirichlet spectrum satisfy the usual Pólya's conjecture eventually, as well we classify all its billiards trajectories. In case of Weyl lunes we derive a generalised two-term Weyl asymptotics with positive terms, and we estimate a lower and an upper bound on the opening angle such that lunes satisfy Pólya conjecture. Moreover, we show that its geodesic orbits are all periodic and each one corresponds to an exact origami of a geodesic $\mathbb{S}^1$ of $\mathbb{S}^n$ folded into the lune at an explicit number of boundary hits.

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Alberto G. Setti (Università degli Studi dell'Insubria)

1983-2025: a journey into the first 40 (+2) years of Marco's mathematics

Abstract: We will retrace, necessarily in broad strokes given his very prolific output, Marco Rigoli's scientific career—from his early works on conformal geometry to his latest research on Einstein-type structures and on minimal and capillary graphs.

Giancarlo Travaglini (Università degli Studi di Milano-Bicocca)

Irregularities of distribution, Convex bodies, and Fourier transforms

Abstract: the term *Irregularities of Distribution* (often referred to as Geometric Discrepancy) was introduced by Klaus Roth in 1954. It refers to the impossibility of achieving a just distribution of N sampling points within the unit cube when tested against a family of convex bodies. In this talk, we use Fourier analytic tools to investigate the relationship between the quality of these sampling points and the geometry of the convex bodies in question.

Laurent Véron (Université de Tours)

Isolated Singularities of Solutions of Equations of Emden-Chandrashekar-Riccati type

Abstract: we study the local properties of solutions of $-\Delta u = e^u - m|\nabla u|^q$ in a punctured domain $\Omega \setminus \{0\}$ of $\mathbf{R}^N$ or in a exterior domain $\mathbf{R}^N \setminus B_{r_0}$ in the range $q > 1$ and $m > 0$. We prove a series of a priori estimates depending on q, and of the sign of $q-2$ and we give a precise description of behaviour of solutions near an isolated singularity or at infinity in $\mathbf{R}^N$. This extends a similar study concerning the Chipot-Weissler equation $-\Delta u = u^p - m|\nabla u|^q$.

Handan Yildirim (Istanbul Üniversitesi)

Legendrian dual surfaces lying in the one-parameter families of the 3-dimensional pseudo-spheres in Lorentz-Minkowski 4-Space

Abstract: in [I], Izumiya gave Legendrian dualities and the duality theorem for the pseudo-spheres in Lorentz-Minkowski space. Later, these Legendrian dualities and the duality theorem were extended in [IY] for one-parameter families of the pseudo-spheres in Lorentz–Minkowski space. In this talk which is based on the joint work with Kentaro Saji given in [SY], we first take into account two extended Legendrian dualities and the extended duality theorem in [IY] and new extended Legendrian dualities and an extended duality theorem given in [SY]. Secondly, we construct the Legendrian dual surfaces, lying in the one-parameter families of the 3-dimensional pseudo-spheres in Lorentz-Minkowski 4-space, of a spacelike curve in the 3-dimensional lightcone. Finally, we give the singularities of these surfaces.

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